

## SEMINARIO DEL DEPARTAMENTO DE MATEMÁTICA APLICADA II

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### Neumann problems with an indefinite weight applied to population genetics

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**Abstract:** In 2002, in the literature of population genetics, a conjecture appeared about the possible uniqueness of nontrivial stationary solutions for a particular class of semilinear parabolic partial differential equations. Thanks to a numerical evidence of multiplicity of positive solutions, we provide a negative answer to that question for two Neumann problems of the form:

$$\begin{cases} p'' + \lambda w(x)f(p) = 0, & \forall x \in [\omega_1, \omega_2], \\ p'(\omega_1) = p'(\omega_2), \\ 0 < p(x) < 1, & \forall x \in [\omega_1, \omega_2], \end{cases}$$

where the weight function  $w(x)$  has indefinite sign with  $\int_{\omega_1}^{\omega_2} w(x) dx < 0$  and the nonlinear term  $f(p)$  is a not concave function of class  $C^2$ , which satisfies  $f(0) = 0 = f(1)$ ,  $f(s) > 0$  for all  $s \in ]0, 1[$  and is such that the map  $s \mapsto f(s)/s$  is (strictly) decreasing.

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**VIERNES 31 de MARZO a las 11:15**  
**Aula T-101 (E. E. Telecomunicación)**

SEMINARIO DEL DEPARTAMENTO DE  
MATEMÁTICA APLICADA II

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Mathematical modelling of group defence and  
infection dynamics in prey

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**Abstract:** Many prey populations have developed group defence strategies to protect themselves from predation. Awe-inspiring pictures easily come to mind from TV documentaries: zebras aggregating to dazzle predators with their stripes; fish forming schools to confuse their predators; or meerkats standing sentry for their clan. The group defence, however, can be undermined if there is an infectious disease spreading in the prey population. What consequences does this have for prey, predators and the stability of the ecological community?

Modelling of such a system seems like a daunting task, as we need to take into account at least the cooperative behaviour of prey, the infection dynamics and the ecological process of predation. It turns out that we come away with a relatively simple mathematical model that consists of three or even two nonlinear ordinary differential equations only. Yet, this model produces complex dynamics and bifurcation sequences, involving Hopf bifurcations, period-doublings to chaos, multistability and attractor crises.

In this talk, I will briefly introduce the biological background, derive the mathematical model and present its analysis. The basic behaviour can be understood in terms of phase plane analysis, but we make use of numerical simulations for non-equilibrium dynamics. We arrive at the conclusion that prey diseases may benefit predators. This is because reduced prey densities weaken their group defence, which in turn allows predators to survive in many situations where they could not without the disease.

This is joint work with Andrew Bate (University of York, UK).

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**VIERNES 31 de MARZO a las 12:10**  
**Aula T-101 (E. E. Telecomunicación)**